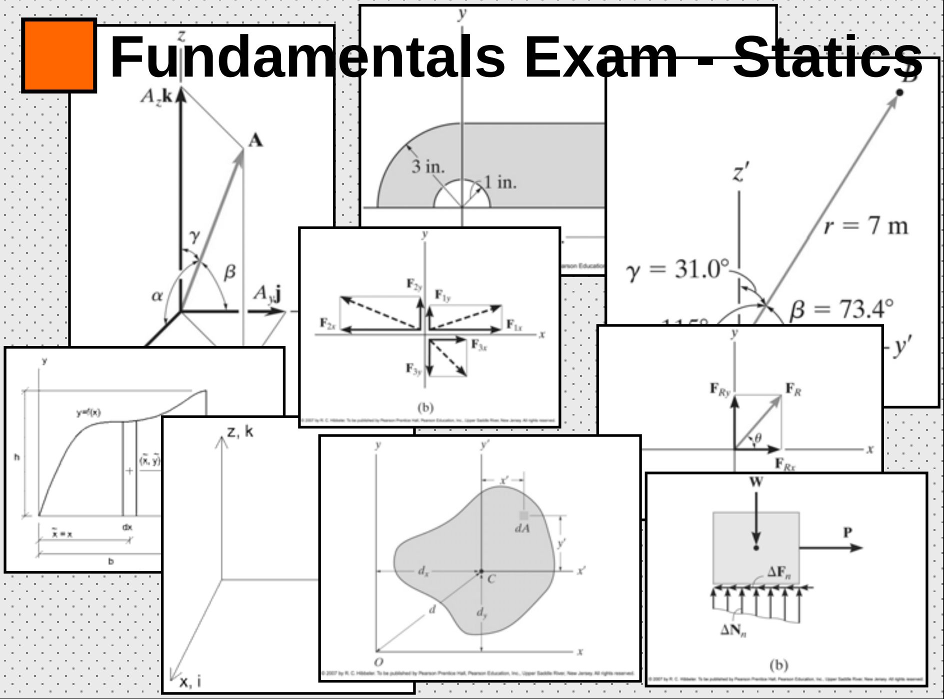
Fundamentals Exam - Statics



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REFERENCES:

- *Fundamentals of Engineering, Supplied-Reference Handbook*, 8th edition; National Council of Examiners for Engineering and Surveying, 2008.
- Hibbeler, R.C.; *Engineering Mechanics Statics*, 11th edition; Pearson Prentice Hall, 2007.

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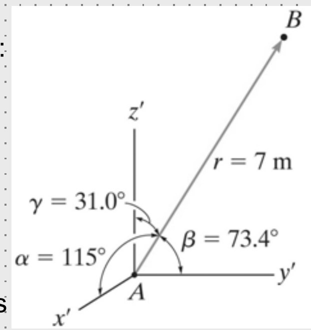
FORCE:

A *force* is a *vector* quantity. It is defined by:



Magnitude - Scalar quantity

Point of Application - two points through which it passes - known as “line of action”

Direction - relative to x, y & z axes

Forces are often represented as ***Vectors***, and can be given in ***Cartesian Vector Form ...***

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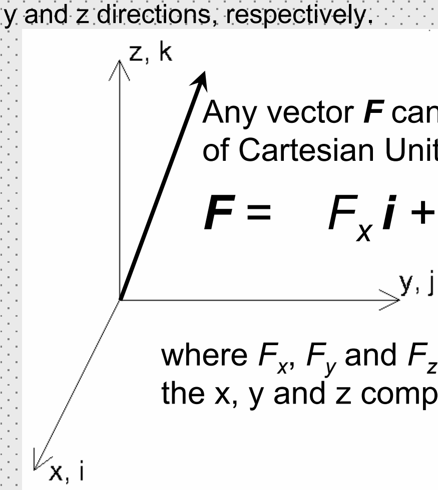
VECTORS - Cartesian Vector Form:

Let ***i***, ***j***, and ***k*** be **Cartesian Unit Vectors** in the x, y and z directions, respectively.



Any vector ***F*** can be written in terms of Cartesian Unit Vectors:

$$\boldsymbol{F} = F_x \boldsymbol{i} + F_y \boldsymbol{j} + F_z \boldsymbol{k}$$

where F_x, F_y and F_z are the magnitudes of the x, y and z components of force

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RESULTANTS (Two Dimensions):

The Cartesian Vector form of a force is:

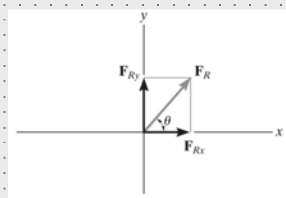
$$\boldsymbol{F} = F_x \boldsymbol{i} + F_y \boldsymbol{j}$$

Resultant force, *F*, w/ ***x*** and ***y*** components known, is found by eqn:

$$F = \sqrt{F_x^2 + F_y^2}$$

Resultant direction of vector w/ respect to the x-axis is found by eqn:

$$\Theta = \tan^{-1} \frac{F_y}{F_x}$$



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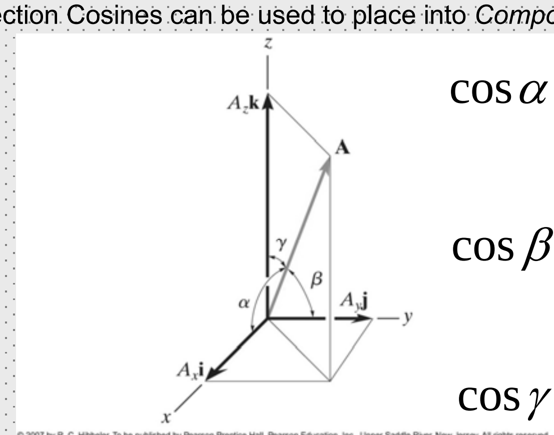
RESOLUTION OF A FORCE:

To describe a single vector as two or more vectors - Also called "*Component Form*" ...

Direction Cosines can be used to place into *Component Form* ...



$$\cos\alpha = \frac{A_x}{A}$$

$$\cos\beta = \frac{A_y}{A}$$

$$\cos\gamma = \frac{A_z}{A}$$

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RESOLUTION OF A FORCE:

The three components become:

$$F_x = F \cos \Theta_x$$
$$F_y = F \cos \Theta_y$$
$$F_z = F \cos \Theta_z$$

or if resultant force $R = \sqrt{x^2 + y^2 + z^2}$ is known:

$$F_x = (x/R)F$$
$$F_y = (y/R)F$$
$$F_z = (z/R)F$$

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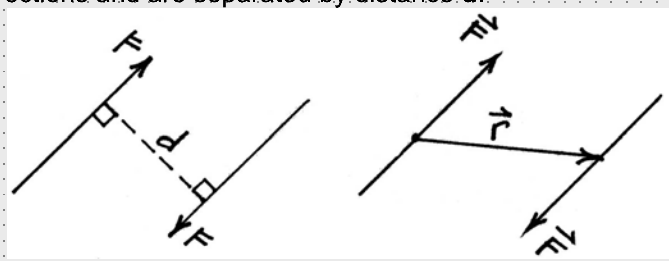
MOMENTS (Couples):

Couple: Two parallel forces with equal magnitudes that act in opposite directions and are separated by distance ***d***.



Magnitude: $M = F\ d$ Vector: $\mathbf{M} = \mathbf{r} \times \mathbf{F}$

NOTE: *The resultant of the forces is ZERO ...*

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MOMENTS - Cross Product:

Vector Moment in Cartesian Form:

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The *determinant* of the matrix will give you the moment about a point in space in Cartesian Vector form.

The eqns for the moment magnitudes about the three axes *for known perpendicular distances* ***x***, ***y*** and ***z*** are:

$M_x = y\,F_z - z\,F_y$ $\quad M_y = z\,F_x - x\,F_z$ $\quad M_z = x\,F_y - y\,F_x$

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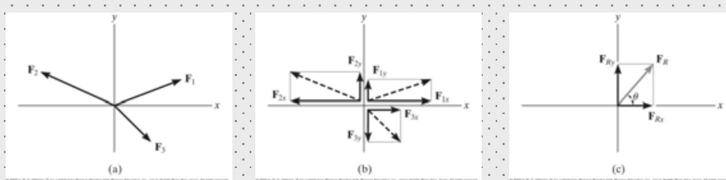
SYSTEMS OF FORCES:

If you have a series of force vectors applied to a system, a single resultant force vector can be determined using the equation:



$$\boldsymbol{F_R} = \Sigma\, \boldsymbol{F_n}$$

In addition, the moment about a point due to multiple forces is:

$$\boldsymbol{M_R} = \Sigma\, (\boldsymbol{r_n} \times \boldsymbol{F_n})$$

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SYSTEMS OF FORCES:

For a body to be in static equilibrium, the forces (applied and reactive) acting on the body must satisfy the equations:

$$\Sigma \boldsymbol{F} = 0 \quad \text{and} \quad \Sigma \boldsymbol{M} = 0$$ ← Vector form

or in component form (6 degrees of freedom)

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$ ← Component

$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$ ← Component

$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$ ← Component

These are the Equilibrium Equations used in Statics ...

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CENTROIDS of Mass, Area, Length and Volume:

Centroid: A point that defines the geometric center of an object.

For a homogenous body, the *centroid* coincides with the *center of gravity* ...

Two general methods to determine centroid are:

Centroid by Integration

Centroid by Composite Bodies

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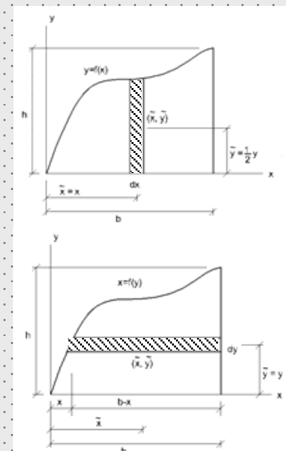
Centroids by Integration:

For finding centroid of an ***area***, we must use the eqns:

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA}$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$



Differential segment through shape used in calculations ...

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Centroids by Integration:

Definition of values in the eqns:

$\bar{x} = \bar{y} =$ Centroidal distance from ***y*** or ***x*** axis

$\tilde{x} = \tilde{y} =$ Distance from ***y***, ***x*** axis to centroid of segment

$dA =$ Differential area of segment

The procedure for finding an area centroid by integration:

Step 1: Choose a differential segment to use. Generally, select a segment that touches one of the reference axes.

Step 2: Define the segment size and moment arm to be used. Draw these on the sketch for reference.

Step 3: Perform the integrations and apply the eqns derived in the text.

Step 4: Ask yourself "*Does the answer make sense?*"

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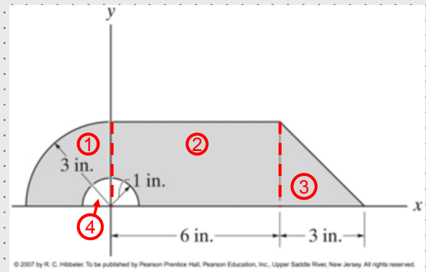
Centroids by Composite Bodies:

For finding centroid of an ***area***, we must use the eqns:

$$\bar{x} = \frac{\Sigma A \tilde{x}}{\Sigma A}$$

$$\bar{y} = \frac{\Sigma A \tilde{y}}{\Sigma A}$$

$$\bar{z} = \frac{\Sigma A \tilde{z}}{\Sigma A}$$



Break shape into 4 elements:

- Quarter circle
- Rectangle
- Triangle
- Semi-circle (Void)

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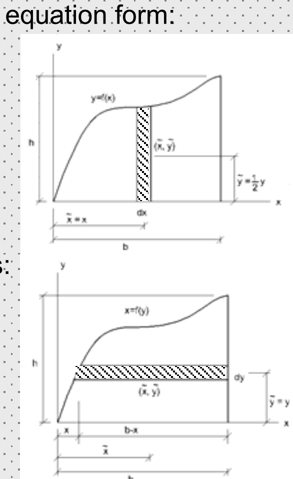
MOMENT OF INERTIA:

The *Moment of Inertia* is defined as the second moment of an area.

This is similar to *Centroids by Integration* is equation form:

$$I_x = \int x^2\, dA$$

$$I_y = \int y^2\, dA$$



The *Polar Moment of Inertia*, ***J*** of an area is:

$$I_z = J = I_x + I_y$$

$$= \int (x^2 + y^2)\, dA$$

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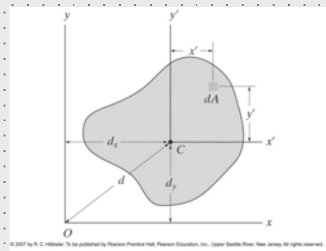
MOMENT of INERTIA by Composite Bodies:

Also known as *Transfer Theorem* or *Parallel-Axis Theorem*.

The *equation for finding the moment of inertia is:*

$$I_x' = I_{x_c} + d_y^2 A$$

$$I_y' = I_{y_c} + d_x^2 A$$



where:

I_x', I_y' = moment of inertia about the new axis

I_{x_c}, I_{y_c} = moment of inertia about the centroidal axis

d_x, d_y = distance between the two axes in question

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RADIUS of GYRATION:

Defined as the distance from a reference axis (***x*** or ***y*** axes, or the *origin*) at which all of the area can be considered to be concentrated to produce the moment of inertia.

In equation form:

$$r_x = \sqrt{I_x / A}$$

$$r_y = \sqrt{I_y / A}$$

$$r_p = \sqrt{J / A}$$

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FRICTION:

Limiting friction is the largest frictional force a body can resist prior to movement.

The equation for *limiting friction* is:

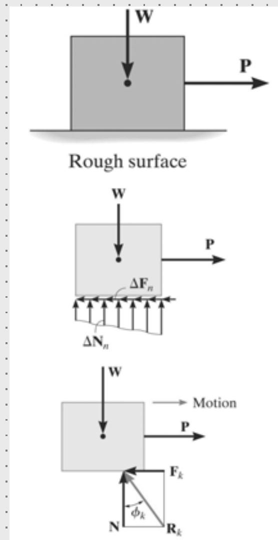
$$F \leq \mu N$$

Where,

F = Friction force

μ = coefficient of static friction

N = normal force between surfaces in contact



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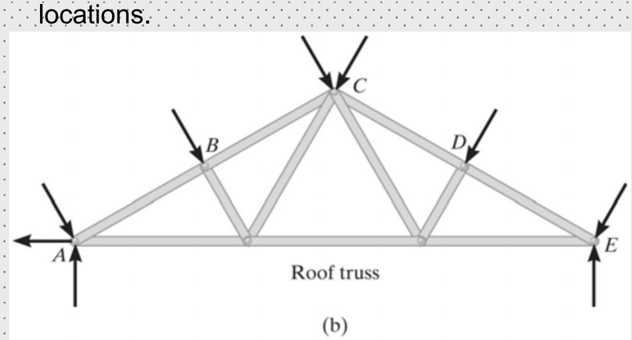
STATICALLY DETERMINATE TRUSS:

Assumptions made when dealing with trusses:

1) Members lie in the same plane (2 - dimensional)
2) Members ends are connected with frictionless pins
3) All external loads (applied and reactive) occur at joint locations.



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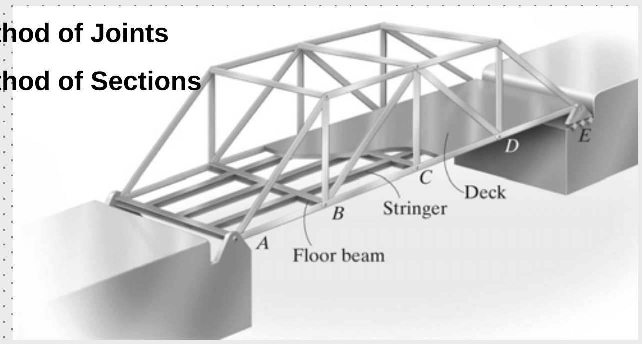
__STATICALLY DETERMINATE TRUSS__:

Truss member forces are determined using the equations:

$$\Sigma \boldsymbol{F} = 0 \quad \text{and} \quad \Sigma \boldsymbol{M} = 0$$

There are two general methods that can be used to analyze a statically determinate truss:

1) **Method of Joints**
2) **Method of Sections**



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Method of Joints:

This method looks at each joint of the truss in determining member forces and uses the eqns:

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

NOTE: *For a truss to be in equilibrium, each joint of the truss must also be in equilibrium …*

Start at a joint with only two unknowns, and work your way across the truss from there. It is typical to solve for support reactions first in this process …

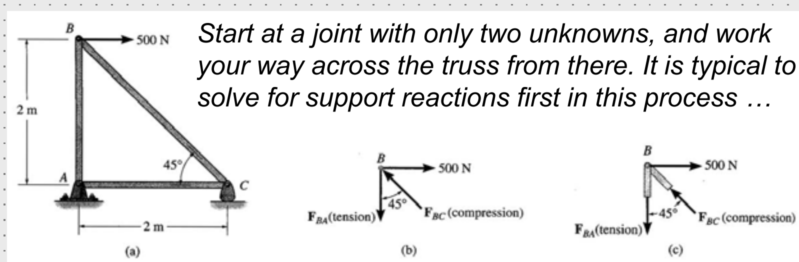

Fig. 6–7

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Method of Sections:

This method looks at sections through the truss in determining member forces and uses the eqns:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M = 0$$

Solve for support reactions first (*in most cases*). Next, cut a section through the members which you are analyzing, and draw a Free Body Diagram of the portion of the truss to the left or right of the section cut. Then apply the three equilibrium equations to determine up to three member forces …

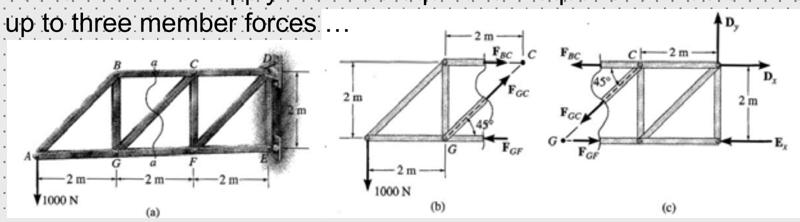

Fig. 6–15

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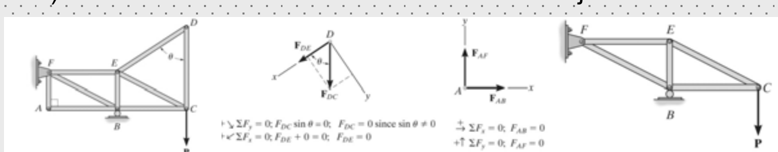
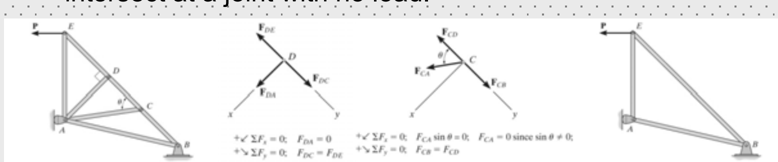
Zero - Force Members:

Two conditions can exist that result in zero-force members:

1) When two non-collinear mbrs intersect at a joint with no load.



2) When two collinear members & a third non-collinear member intersect at a joint with no load.



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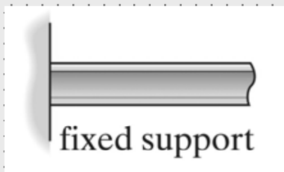
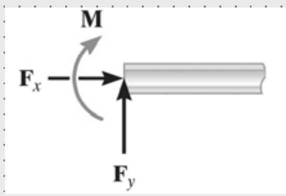
<u>SUPPORT REACTIONS</u>:
Different support conditions result in certain support reactions. Refer to *Hibbeler's* **Table 5-1** for 2-D reactions:





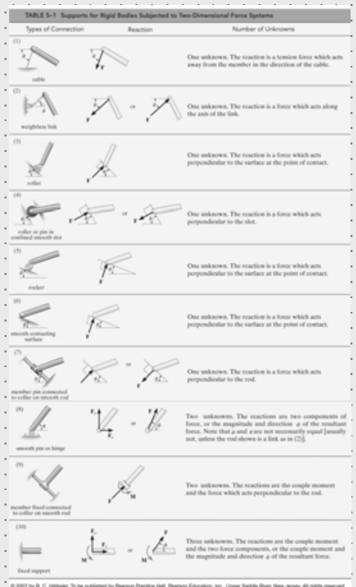


TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1) cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2) weightless link		One unknown. The reaction is a force which acts along the axis of the link.
(3) roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(5) rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6) smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7) member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.
(8) smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9) member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10) fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

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<u>SUPPORT REACTIONS</u>:
Different support conditions result in certain support reactions.
Refer to *Hibbeler's* **Table 5-2** for 3-D reactions:

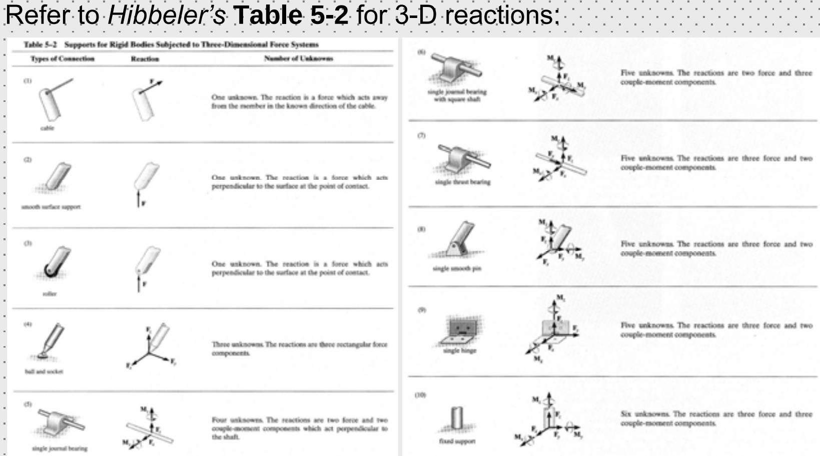

Table 5-2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1) cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2) smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3) roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) ball and socket		Three unknowns. The reactions are three rectangular force components.
(5) single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft.
(6) single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components.
(7) single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components.
(8) single smooth pin		Five unknowns. The reactions are three force and two couple-moment components.
(9) single hinge		Five unknowns. The reactions are three force and two couple-moment components.
(10) fixed support		Six unknowns. The reactions are three force and three couple-moment components.

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Practice Questions:

- **Refer to Appendix C of Hibbeler Text:**

  Engineering Mechanics Statics, 4th edition

55 problems to work through - partial solutions are given immediately after the problems ...

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QUESTIONS